

## Complex Analysis with Applications

In this exam, we are going to test the following subjects: Elementary functions: exponential functions, log functions, power functions, sin function and their associated mapping properties. Path integral and the evaluation of path integral by parametrization. The Cauchy integral formulas will also be tested in the exam. Pay attention, what we mean Cauchy integral formula includes the formulas for higher order derivatives of a holomorphic function. Here in the following are some sample problems.

### Problem 1:

Let  $\gamma$  be the positively oriented circle with radius 1 and center  $i$ . Stating clearly any theorems you use, evaluate the following contour integrals:

$$(a) \int_{\gamma} \bar{z} dz; \quad (b) \int_{\gamma} \frac{1}{z^2 + 2} dz; \quad (c) \int_{\gamma} \frac{1}{(z^2 + 2)^3} dz; \quad (d) \int_{\gamma} \frac{1}{z^2 - 2} dz.$$

**Problem 2:** Find all solutions to the equation

$$e^z = 1 + i.$$

**Problem 3:** Find bijective elementary mapping which maps the region  $A$  to  $B$ . Here  $A$  is the region which contains all points on  $\mathbb{C}$  without the segment

$$\{ti : -1 \leq t \leq 1\}.$$

$B$  is the region between the two branches of the hyperbola

$$2x^2 - 2y^2 = 1.$$